## Simple Physics of Golf Ball Flight

Considerations for estimating the flight speed, distance, and time of flight of a golf ball after impact with a driver (more precisely, the driver head).

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For those in a hurry, the answers regarding ball speed can be found in equations 15 and 16 . Similarly, for flight distance, refer to equations 18 and 20, and for flight duration, please proceed to the end.

## Calculation of Golf Ball and Club Head Speed Shortly After Impact

The following considerations are based on the well-known conservation principles of energy $E$ and momentum $p$. The initial situation is clear: the golf ball is at rest on the ground (ball speed $v_{B}=0$ ) and is struck by a golf club (driver), more precisely by its club head. Both the golf ball and the club head are assumed to be perfectly rigid (elastic collision). The following formulas address the one-dimensional case, where the club head comes horizontally from the left and strikes the golf ball, which then flies horizontally to the right after impact. This scenario would be observed, for example, when a golfer plays the golf ball off the tee with a driver using their right hand, and the observer, or a technical device like a video camera, views the player frontally. The following graphic illustrates these three phases using an example of a golf swing by Rory McIllroy (YouTube).


Just before impact, the following applies to the club head:

$$
\begin{align*}
p_{0} & =m_{S} \cdot v_{0}  \tag{1}\\
E_{0} & =\frac{1}{2} m_{S} \cdot v_{0}^{2} \tag{2}
\end{align*}
$$

Here, $m_{S}$ is the mass of the golf club head and $v_{0}$ is the speed of the club head at impact. $E_{0}$ is the kinetic energy of the club head, and $p_{0}$ is its momentum. The energy $E_{B}$ and momentum $p_{B}$ of the golf ball are both zero before impact.
For the total momentum $p$ and total energy $E$ of the golf ball and club head before impact, we have:

$$
\begin{align*}
& p=p_{0}  \tag{3}\\
& E=E_{0} \tag{4}
\end{align*}
$$

Just after impact, the total momentum and total energy are given by:

$$
\begin{gather*}
p=m_{S} \cdot v_{S}+m_{B} \cdot v_{B}  \tag{5}\\
E=\frac{1}{2} m_{S} \cdot v_{S}^{2}+\frac{1}{2} m_{B} \cdot v_{B}^{2} \tag{6}
\end{gather*}
$$

The momentum of the golf ball with mass $m_{B}$ and velocity $v_{B}$ is added. The total kinetic energy $E$ is divided after impact into the kinetic energy of the golf ball $E_{B}=\frac{1}{2} m_{B} v_{B}^{2}$ and that of the club head $E_{S}=\frac{1}{2} m_{S} v_{S}^{2}$. The velocity of the club head $v_{S}$ is less after impact than before, i.e., $v_{S}<v_{0}$. The same applies to the energy $E_{S}<E_{0}$.

Now, equations 1. 3. and 5 can be combined as follows:

$$
\begin{equation*}
m_{S} \cdot v_{0}=m_{S} \cdot v_{S}+m_{B} \cdot v_{B} \tag{7}
\end{equation*}
$$

Similarly, combining equations 2, 4, and 6 yields:

$$
\begin{equation*}
\frac{1}{2} m_{S} \cdot v_{0}^{2}=\frac{1}{2} m_{S} \cdot v_{S}^{2}+\frac{1}{2} m_{B} \cdot v_{B}^{2} \tag{8}
\end{equation*}
$$

Equation 8 can be conveniently rearranged. The idea for this can be found, for example, in the book Physics for Scientists and Engineers by Serway and Jewett, pages 235-237 of the 7th edition from 2008. By eliminating the factor $1 / 2$ and rewriting, we get:

$$
\begin{equation*}
m_{S} \cdot\left(v_{0}^{2}-v_{S}^{2}\right)=m_{B} \cdot v_{B}^{2} \tag{9}
\end{equation*}
$$

The left-hand side can be factorized. Note: $\left(a^{2}-b^{2}\right)=(a-b)(a+b)$ :

$$
\begin{equation*}
m_{S} \cdot\left(v_{0}-v_{S}\right)\left(v_{0}+v_{S}\right)=m_{B} \cdot v_{B}^{2} \tag{10}
\end{equation*}
$$

To see the simplification idea, equation 7 is rewritten as follows:

$$
\begin{equation*}
m_{S} \cdot\left(v_{0}-v_{S}\right)=m_{B} \cdot v_{B} \tag{11}
\end{equation*}
$$

The simplification is achieved by dividing equation 10 by equation 11

$$
\begin{gather*}
\left(v_{0}+v_{S}\right)=v_{B}  \tag{12}\\
v_{0}=v_{B}-v_{S} \tag{13}
\end{gather*}
$$

This equation, along with equation 11, makes it very easy to calculate the speeds of the golf ball and club head shortly after impact:

$$
\begin{align*}
& v_{S}=\frac{m_{S}-m_{B}}{m_{S}+m_{B}} \cdot v_{0}  \tag{14}\\
& v_{B}=\frac{2 m_{S}}{m_{S}+m_{B}} \cdot v_{0} \tag{15}
\end{align*}
$$

During the flight, the speed of the ball $v_{B}$ typically decreases.
Now, let's verify the results with real-world numbers. The golf ball weighs a maximum of $m_{B}=45.93 \mathrm{~g}$. A driver head has a mass of approximately $m_{S}=200 \mathrm{~g}$. For the club head speeds $v_{0}$, a very high value can be taken, for example, from Bryson DeChambeau, e.g., $v_{0}=240.4 \mathrm{~km} / \mathrm{h}(149.4 \mathrm{mph})$. Using equation 15 , we can calculate the speed of the golf ball shortly after impact as:

$$
\begin{equation*}
v_{B}=391 \mathrm{~km} / \mathrm{h}=243 \mathrm{mph} \tag{16}
\end{equation*}
$$

In this case, the calculated ball speed is approximately $10 \%$ higher than the measured value of 221.5 mph using Trackman.

Since the golf ball deforms upon impact with the driver head, and additionally, the club face of a driver can act like a tensioned membrane (trampoline effect), and also other factors (dimples, spin) play a role, this is pretty close to the maximum golf ball speed achievable in reality. If the mass of the golf ball is negligible compared to the mass of the club head, the theoretically maximum speed of the golf ball is given by:

$$
\begin{equation*}
v_{B}=2 \cdot v_{0} \tag{17}
\end{equation*}
$$

Using the given numbers, the club head speed after impact according to Equation 14 is $v_{S}=150.6 \mathrm{~km} / \mathrm{h}$ ( 93.6 mph ). The club head has been significantly decelerated.
For recreational golfers with club head speeds before impact typically below $145 \mathrm{~km} / \mathrm{h}(90 \mathrm{mph})$, the maximum ball speed accordingly is approximately $236 \mathrm{~km} / \mathrm{h}(147 \mathrm{mph})$. This is close to the speeds achieved by the best tennis players during a serve.

## Calculation of Golf Ball Flight Distance

To estimate the flight distance of the golf ball, both the horizontal and vertical directions need to be considered. This requires departing from the one-dimensional approach used for calculating speeds and involves a bit more mathematics. It's helpful to refer to the Wikipedia article on Projectile motion. There, the maximum range from the ground under neglecting air resistance is given by:

$$
\begin{equation*}
R_{\max }=\frac{v_{B}^{2}}{g} \cdot \sin 2 \beta \tag{18}
\end{equation*}
$$

where $g$ denotes the acceleration due to gravity $\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)$ and $\beta$ represents the launch angle relative to the horizontal. The sine function reaches its maximum value at $\beta=45$ degrees, $\operatorname{so} \sin (2 \cdot 45)=1$.

The range is inversely proportional to $g$, thus, for example, on the moon with the same velocity $v_{B}$, it would be about 6 times as far $\left(g_{\text {Earth }}=9.81 \mathrm{~m} / \mathrm{s}^{2}, g_{\text {Moon }}=1.66 \mathrm{~m} / \mathrm{s}^{2}\right)$.

However, Equation 18 only applies when air resistance is neglected. When it is considered, an optimal launch angle $\beta$ of approximately 20 degrees is obtained, and the parabolic trajectory bends downward rapidly after reaching its maximum, meaning the ball falls almost vertically to the ground. The range is reduced to about $64 \%(\sin (2 * 20$ degrees $)=0.64)$ of the maximum flight distance. The Wikipedia graphic illustrates flight trajectories for various launch angles.
With the previously used values, $v_{B}=391 \mathrm{~km} / \mathrm{h}$ and $\beta=20$ degrees, the maximum ball flight distance according to Equation 19 is thus:

$$
\begin{equation*}
R_{\max }(\beta=20 \text { degrees })=772.9 \mathrm{~m} \tag{19}
\end{equation*}
$$

This applies, mind you, only to the assumed projectile motion. Looking at the launch angles achieved in professional golf with a driver (Launch Angle), these according to Trackman average around 10 degrees for the PGA Tour

Hence, $50 \%$ lower than the optimal angle of a projectile motion with air resistance. Accordingly, the calculated maximum flight distance is shorter. If we take the measured launch angle of 8.5 degrees from the data of Bryson DeChambeau, as well as the previously calculated ball speed $v_{B}=391 \mathrm{~km} / \mathrm{h}$, the flight distance is calculated as:

$$
\begin{equation*}
R_{\max }(\beta=8.5 \text { degrees })=351.6 \mathrm{~m} \tag{20}
\end{equation*}
$$

This surprisingly matches well with the measured flight distance of 344.1 m .
The actual ball flight is significantly more complex; accordingly, the flight distances in reality are different and considerably shorter. For those interested in delving deeper, one can search for publications using keywords like "golf ball aerodynamics". A taste of the physics behind realistic golf ball flight trajectories can be found in the freely accessible eBook by Dr. Eugen Willerding in Chapter 4.15.

Exemplary are typical PGA Tour trajectories, as determined by Trackman, shown in the following graphic ( 1 Yard $\approx 0.9 \mathrm{~m}$ ).


## Flight Duration of the Golf Ball

Here, the question is how long the golf ball remains in the air, from the tee shot until it lands on the ground. In English, this time is often referred to as hang time. Looking at the PGA Tour statistics for this, these times average around 6.5 seconds, ranging approximately between 6 and 7 seconds.

Please remember, these considerations and calculations are based on simplifying assumptions.

